SADLER MATHEMATICS SPECIALIST UNIT 2

WORKED SOLUTIONS

Chapter 10 Matrices

Exercise 10A

Question 1

$A_{4\times 2}$ $B_{2\times 4}$ $C_{4\times 1}$ $D_{4\times 3}$ $E_{2\times 2}$ $F_{1\times 3}$ $G_{3\times 2}$ $H_{4\times 4}$

Question 2

a 4

b -4

c 7

d 7

e 3

f 0

Question 3

a Matrices of different sizes – cannot be determined

b $\begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 1 & -5 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -9 \end{bmatrix}$

c $\begin{bmatrix} 2 & -3 \\ 1 & -5 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ 1 & -1 \end{bmatrix}$

d $2\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ -4 \end{bmatrix}$

e $3\begin{bmatrix} 3 & -1 \\ 2 & 4 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ 6 & 12 \\ 0 & 9 \end{bmatrix}$

f Matrices of different sizes – cannot be determined

g $2\begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 0 & -8 \end{bmatrix}$

h $2\begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 1 & -5 \end{bmatrix} = \begin{bmatrix} 0 & 7 \\ -1 & -3 \end{bmatrix}$

Question 4

a $\begin{bmatrix} 3 & 2 & -1 \\ 1 & 4 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 3 & -1 \\ 1 & 3 & 3 \end{bmatrix}$

b $\begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 2 & -1 \\ 1 & 4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 \\ -1 & -5 & -3 \end{bmatrix}$

c $3\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 3 \\ 6 & 3 & 6 \end{bmatrix}$

d

$$3\begin{bmatrix} 3 & 2 & -1 \\ 1 & 4 & 3 \end{bmatrix} - 2\begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 9 & 6 & -3 \\ 3 & 12 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 4 & -3 \\ 3 & 14 & 9 \end{bmatrix}$$

Question 5

a Matrices of different sizes – cannot be determined

b $3\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 3 & 9 \end{bmatrix}$

c $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix} + 2\begin{bmatrix} 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 3 & 11 \end{bmatrix}$

d Matrices of different sizes – cannot be determined

Question 6

a Matrices of different sizes – cannot be determined

b $$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 1 & 3 & -1 \\ 2 & 1 & 4 & 3 \\ 1 & 5 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 4 & 3 & 0 \\ 2 & 2 & 6 & 6 \\ 1 & 5 & 3 & 4 \end{bmatrix}$$

c $$2\begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & -3 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 8 \\ 4 & 2 & -6 \\ 0 & 2 & 4 \\ 2 & 0 & 0 \end{bmatrix}$$

d $$5\begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 1 & 3 & -1 \\ 2 & 1 & 4 & 3 \\ 1 & 5 & 2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 15 & 0 & 5 \\ 0 & 5 & 10 & 15 \\ 0 & 0 & 5 & 20 \end{bmatrix} - \begin{bmatrix} 5 & 1 & 3 & -1 \\ 2 & 1 & 4 & 3 \\ 1 & 5 & 2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 14 & -3 & 6 \\ -2 & 4 & 6 & 12 \\ -1 & -5 & 3 & 20 \end{bmatrix}$$

Question 7

a No

b No

c Yes

d Yes

e Yes

f No

g Yes

h No

Question 8

Yes, as addition is commutative

Question 9

Yes, as addition is associative

Question 10

$$3A - 2C = B$$
$$2C = 3A - B$$
$$C = \frac{1}{2}[3A - B]$$

$$3\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & -7 & 12 \\ 1 & 0 & 13 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 4 & -6 \\ 2 & 0 & -4 \end{bmatrix}$$
$$C = \frac{1}{2}\begin{bmatrix} 2 & 4 & -6 \\ 2 & 0 & -4 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 & -3 \\ 1 & 0 & -2 \end{bmatrix}$$

Question 11

a Addition of all four matrices produces :

$$\begin{array}{r c}
 & \begin{array}{ccc} \text{P} & \text{A} & \text{B} \end{array} \\
\begin{array}{r} \text{Alan} \\ \text{Bob} \\ \text{Dave} \\ \text{Mark} \\ \text{Roger} \end{array} & \begin{bmatrix} 40 & 20 & 4 \\ 37 & 15 & 14 \\ 47 & 49 & 9 \\ 39 & 21 & 3 \\ 39 & 19 & 16 \end{bmatrix}
\end{array}$$

b $$\frac{1}{4}\begin{bmatrix} 40 & 20 & 4 \\ 37 & 15 & 14 \\ 47 & 19 & 9 \\ 39 & 21 & 3 \\ 39 & 19 & 16 \end{bmatrix} = \begin{bmatrix} 10 & 5 & 1 \\ 9.25 & 3.75 & 3.5 \\ 11.75 & 4.75 & 2.25 \\ 9.75 & 5.25 & 0.75 \\ 9.75 & 4.75 & 4 \end{bmatrix}$$

Question 12

$$\begin{bmatrix} 3100 & 550 & 1040 & 820 & 2250 \\ 1640 & 420 & 720 & 480 & 1480 \\ 2850 & 520 & 1320 & 640 & 1250 \\ 1240 & 300 & 800 & 360 & 960 \end{bmatrix} + \begin{bmatrix} 2500 & 1200 & 1280 & 950 & 2000 \\ 1200 & 850 & 650 & 540 & 1240 \\ 2200 & 950 & 1500 & 640 & 1450 \\ 950 & 640 & 720 & 480 & 820 \end{bmatrix} = \begin{bmatrix} 5600 & 1750 & 2320 & 1770 & 4250 \\ 2840 & 1270 & 1370 & 1020 & 2720 \\ 5050 & 1470 & 2820 & 1280 & 2700 \\ 2190 & 940 & 1520 & 840 & 1780 \end{bmatrix}$$

$$1.1\begin{bmatrix} 5600 & 1750 & 2320 & 1770 & 4250 \\ 2840 & 1270 & 1370 & 1020 & 2720 \\ 5050 & 1470 & 2820 & 1280 & 2700 \\ 2190 & 940 & 1520 & 840 & 1780 \end{bmatrix} = \begin{bmatrix} 6160 & 1925 & 2552 & 1947 & 4675 \\ 3124 & 1397 & 1507 & 1122 & 2992 \\ 5555 & 1617 & 3102 & 1408 & 2970 \\ 2409 & 1034 & 1672 & 924 & 1958 \end{bmatrix}$$

Question 13

$a_{11} = 2 \times 1 + 1 = 3$
$a_{12} = 2 \times 1 + 2 = 4$
$a_{13} = 2 \times 1 + 3 = 5$
$a_{21} = 2 \times 2 + 1 = 5$
$a_{22} = 2 \times 2 + 2 = 6$
$a_{23} = 2 \times 2 + 3 = 7$
$a_{31} = 2 \times 3 + 1 = 7$
$a_{32} = 2 \times 3 + 1 = 8$
$a_{33} = 2 \times 3 + 3 = 9$

$$\text{A} = \begin{bmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{bmatrix}$$

Question 14

$a_{11} = 1^1 = 1$
$a_{12} = 1^2 = 1$
$a_{13} = 1^3 = 1$
$a_{14} = 1^4 = 1$
$a_{21} = 2^1 = 2$
$a_{22} = 2^2 = 4$
$a_{23} = 2^3 = 8$
$a_{24} = 2^4 = 16$
$a_{31} = 3^1 = 3$
$a_{32} = 3^2 = 9$
$a_{33} = 3^3 = 27$
$a_{34} = 3^4 = 81$

$$\text{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 8 & 16 \\ 3 & 9 & 27 & 81 \end{bmatrix}$$

Exercise 10B

Question 1

$$\begin{bmatrix}1\times2+2\times1 & 1\times3+2\times3\end{bmatrix}$$
$$=\begin{bmatrix}4 & 9\end{bmatrix}$$

Question 2

Cannot be determined – number of columns in matrix 1 does not equal the number of rows in matrix 2.

$$2\times\boxed{2}\times\boxed{1}\times2$$

Question 3

$$\begin{bmatrix}2\times1+(-1)\times0 & 2\times4+(-1)\times(-2)\\ 1\times1+0\times0 & 1\times4+0\times(-2)\end{bmatrix}$$
$$=\begin{bmatrix}2 & 10\\ 1 & 4\end{bmatrix}$$

Question 4

$$\begin{bmatrix}3\times1+1\times4\end{bmatrix}$$
$$=\begin{bmatrix}7\end{bmatrix}$$

Question 5

$$\begin{bmatrix}1\times3 & 1\times1\\ 4\times3 & 4\times1\end{bmatrix}$$
$$=\begin{bmatrix}3 & 1\\ 12 & 4\end{bmatrix}$$

Question 6

$$\begin{bmatrix} 2\times 2+(-3)\times(-3) & 2\times 1+(-3)\times 2 \\ (-1)\times 2+4\times(-3) & (-1)\times 1+4\times 2 \end{bmatrix}$$
$$=\begin{bmatrix} 13 & -4 \\ -14 & 7 \end{bmatrix}$$

Question 7

$$\begin{bmatrix} 1\times 2+0\times 1 & 1\times 3+0\times(-1) \\ 0\times 2+1\times 1 & 0\times 3+1\times(-1) \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$$

Question 8

$$\begin{bmatrix} 1\times 1+4\times 0 & 1\times 0+4\times 1 \\ (-1)\times 1+3\times 0 & (-1)\times 0+3\times 1 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 4 \\ -1 & 3 \end{bmatrix}$$

Question 9

$$\begin{bmatrix} 0\times 2+0\times 4 & 0\times 1+0\times 5 \\ 0\times 2+0\times 4 & 0\times 1+0\times 5 \end{bmatrix}$$
$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Question 10

$$\begin{bmatrix} 3\times 2+1\times(-5) & 3\times(-1)+1\times 3 \\ 5\times 2+2\times(-5) & 5\times(-1)+2\times 3 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Question 11

$$\begin{bmatrix} 8\times 2+(-5)\times 3 & 8\times 5+(-5)\times 8 \\ (-3)\times 2+2\times 3 & (-3)\times 5+2\times 8 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Question 12

$$\begin{bmatrix} 3\times 0.5+1\times(-0.5) & 3\times(-0.5)+1\times 1.5 \\ 1\times 0.5+1\times(-0.5) & 1\times(-0.5)+1\times 1.5 \end{bmatrix}$$
$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Question 13

$$\begin{bmatrix} 1\times 2+2\times 1+1\times 2+2\times 1 \end{bmatrix}$$
$$=\begin{bmatrix} 8 \end{bmatrix}$$

Question 14

$$\begin{bmatrix} 1\times 1+0\times 3+1\times 2+0\times 1 & 1\times 0+0\times(-1)+1\times 2+0\times 4 & 1\times 1+0\times 0+1\times 2+0\times 1 \\ 0\times 1+1\times 3+0\times 2+1\times 1 & 0\times 0+1\times(-1)+0\times 2+1\times 4 & 0\times 1+1\times 0+0\times 2+1\times 1 \end{bmatrix}$$
$$=\begin{bmatrix} 3 & 2 & 3 \\ 4 & 3 & 1 \end{bmatrix}$$

Question 15

$$\begin{bmatrix} 1\times1+0\times5 & 1\times0+0\times1 & 1\times5+0\times(-1) \\ 0\times1+2\times5 & 0\times0+2\times1 & 0\times5+2\times(-1) \\ 1\times1+1\times5 & 1\times0+1\times1 & 1\times5+1\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 10 & 2 & -2 \\ 6 & 1 & 4 \end{bmatrix}$$

Question 16

$$\begin{bmatrix} 1\times1+3\times4+1\times(-3) & 1\times2+3\times1+1\times(-2) \\ 3\times1+0\times4+(-2)\times(-3) & 3\times2+0\times1+\times(-2)\times(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 3 \\ 9 & 10 \end{bmatrix}$$

Question 17

$$\begin{bmatrix} 1\times1+2\times2+3\times3 \\ 4\times1+5\times2+6\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 14 \\ 32 \end{bmatrix}$$

Question 18

$$\begin{bmatrix} 2\times1+1\times0+0\times3 & 2\times1+1\times2+0\times1 & 2\times(-1)+1\times3+0\times4 \\ (-1)\times1+3\times0+2\times3 & (-1)\times1+3\times2+2\times1 & (-1)\times(-1)+3\times3+2\times4 \\ 0\times1+2\times0+4\times3 & 0\times1+2\times2+4\times1 & 0\times(-1)+2\times3+4\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 1 \\ 5 & 7 & 18 \\ 12 & 8 & 22 \end{bmatrix}$$

Question 19

a

$$\begin{bmatrix} 1\times0+0\times2+(-1)\times0 & 1\times1+0\times1+(-1)\times(-1) & 1\times2+0\times0+(-1)\times1 \\ 2\times0+0\times2+1\times0 & 2\times1+0\times1+1\times(-1) & 2\times2+0\times0+1\times1 \\ 0\times0+1\times2+1\times0 & 0\times1+1\times1+1\times(-1) & 0\times2+1\times0+1\times1 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 2 & 1 \\ 0 & 1 & 5 \\ 2 & 0 & 1 \end{bmatrix}$$

b

$$\begin{bmatrix} 0\times1+1\times2+2\times0 & 0\times0+1\times0+2\times1 & 0\times(-1)+1\times1+2\times1 \\ 2\times1+1\times2+0\times0 & 2\times0+1\times0+0\times1 & 2\times(-1)+1\times1+0\times1 \\ 0\times1+(-1)\times2+1\times0 & 0\times0+(-1)\times0+1\times1 & 0\times(-1)+(-1)\times1+1\times1 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & 2 & 3 \\ 4 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

c

$$\begin{bmatrix} 1\times1+0\times2+(-1)\times0 & 1\times0+0\times0+(-1)\times1 & 1\times(-1)+0\times1+(-1)\times1 \\ 2\times1+0\times2+1\times0 & 2\times0+0\times0+1\times1 & 2\times(-1)+0\times1+1\times1 \\ 0\times1+1\times2+1\times0 & 0\times0+1\times0+1\times1 & 0\times(-1)+1\times1+1\times1 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & -1 \\ 2 & 1 & 2 \end{bmatrix}$$

d

$$\begin{bmatrix} 0\times0+1\times2+2\times0 & 0\times1+1\times1+2\times(-1) & 0\times2+1\times0+2\times1 \\ 2\times0+1\times2+0\times0 & 2\times1+1\times1+0\times(-1) & 2\times2+1\times0+0\times1 \\ 0\times0+(-1)\times2+1\times0 & 0\times1+(-1)\times1+1\times(-1) & 0\times2+(-1)\times0+1\times1 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & -1 & 2 \\ 2 & 3 & 4 \\ -2 & -2 & 1 \end{bmatrix}$$

Question 20

By considering **19 a** and **19 b** it is clear that matrix multiplication is not commutative as the matrices produced when the order of multiplication is reversed are not the same.

Question 21

a

$$\begin{aligned}\text{AB} &= \begin{bmatrix} 1\times 3+2\times 0 & 1\times 1+2\times(-1) \\ (-1)\times 3+0\times 0 & (-1)\times 1+0\times(-1)\end{bmatrix} \\ &= \begin{bmatrix} 3 & -1 \\ -3 & -1\end{bmatrix}\end{aligned}$$

$$\begin{aligned}\text{(AB)C} &= \begin{bmatrix} 3\times 1+(-1)\times(-1) & 3\times 2+(-1)\times 1 \\ (-3)\times 1+(-1)\times(-1) & (-3)\times 2+(-1)\times 1\end{bmatrix} \\ &= \begin{bmatrix} 4 & 5 \\ -2 & -7\end{bmatrix}\end{aligned}$$

$$\begin{aligned}\text{BC} &= \begin{bmatrix} 3\times 1+1\times(-1) & 3\times 2+1\times 1 \\ 0\times 1+(-1)\times(-1) & 0\times 2+(-1)\times 1\end{bmatrix} \\ &= \begin{bmatrix} 2 & 7 \\ 1 & -1\end{bmatrix}\end{aligned}$$

$$\begin{aligned}\text{A(BC)} &= \begin{bmatrix} 1 & 2 \\ -1 & 0\end{bmatrix}\begin{bmatrix} 2 & 7 \\ 1 & -1\end{bmatrix} \\ &= \begin{bmatrix} 1\times 2+2\times 1 & 1\times 7+2\times(-1) \\ (-1)\times 2+0\times 1 & (-1)\times 7+0\times(-1)\end{bmatrix} \\ &= \begin{bmatrix} 4 & 5 \\ -2 & -7\end{bmatrix}\end{aligned}$$

(AB)C=A(BC)

b

$$\begin{aligned}AB &= \begin{bmatrix}1\times1+2\times2 & 1\times0+2\times1 & 1\times(-1)+2\times1\end{bmatrix}\\ &= \begin{bmatrix}5 & 2 & 1\end{bmatrix}\end{aligned}$$

$$\begin{aligned}(AB)C &= \begin{bmatrix}5 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 2\\ 1 & 1\end{bmatrix}\\ &= \begin{bmatrix}5\times1+2\times(-1)+1\times1 & 5\times0+2\times2+1\times1\end{bmatrix}\\ &= \begin{bmatrix}4 & 5\end{bmatrix}\end{aligned}$$

$$BC = \begin{bmatrix}1\times1+0\times(-1)+(-1)\times1 & 1\times0+0\times2+(-1)\times1\\ 2\times1+1\times(-1)+1\times1 & 2\times0+1\times2+1\times1\end{bmatrix}$$

$$= \begin{bmatrix}0 & -1\\ 2 & 3\end{bmatrix}$$

$$\begin{aligned}A(BC) &= \begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}0 & -1\\ 2 & 3\end{bmatrix}\\ &= \begin{bmatrix}1\times0+2\times2 & 1\times(-1)+2\times3\end{bmatrix}\\ &= \begin{bmatrix}4 & 5\end{bmatrix}\end{aligned}$$

(AB)C=A(BC)

Question 22

a $\mathrm{A(B+C)} = \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}\left(\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}\right)$

$$= \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1+1\times(-1) & 2\times2+1\times4 \\ 4\times1+0\times(-1) & 4\times2+0\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 8 \\ 4 & 8 \end{bmatrix}$$

$$\mathrm{AB+AC} = \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times(-1)+1\times0 & 2\times1+1\times1 \\ 4\times(-1)+0\times0 & 4\times1+0\times1 \end{bmatrix} + \begin{bmatrix} 2\times2+1\times(-1) & 2\times1+1\times3 \\ 4\times2+0\times(-1) & 4\times1+0\times3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 3 \\ -4 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ 8 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 8 \\ 4 & 8 \end{bmatrix}$$

b $\mathrm{A(B+C)} = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \end{bmatrix}\right)$

$$= \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times 2 + 0\times 6 \\ (-3)\times 2 + 1\times 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\mathrm{AB+AC} = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times 3 + 0\times 2 \\ (-3)\times 3 + 1\times 2 \end{bmatrix} + \begin{bmatrix} 2\times(-1) + 0\times 4 \\ (-3)\times(-1) + 1\times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ -7 \end{bmatrix} + \begin{bmatrix} -2 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

Question 23

$$(k\text{A})\text{B} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix}$$
$$= \begin{bmatrix} kae + kbg & kaf + kbh \\ kce + kdg & kcf + kdh \end{bmatrix}$$

$$\text{A}(k\text{B}) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} ke & kf \\ kg & kh \end{bmatrix}$$
$$= \begin{bmatrix} kae + kbg & kaf + kbh \\ kce + kdg & kcf + kdh \end{bmatrix}$$

$$k(\text{AB}) = k\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix}$$
$$= k\begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$
$$= \begin{bmatrix} kae + kbg & kaf + kbh \\ kce + kdg & kcf + kdh \end{bmatrix}$$

Statement is true

Question 24

a AB cannot be formed

$3\times\boxed{2}\times\boxed{3}\times2$ - number of columns from matrix A $\neq$ number of rows in matrix B

b BA cannot be formed

$3\times\boxed{2}\times\boxed{3}\times2$ - number of columns from matrix B $\neq$ number of rows in matrix A

c BC: $3\times\boxed{2}\times\boxed{2}\times3\Rightarrow3\times3$

d CB: $2\times\boxed{3}\times\boxed{3}\times2\Rightarrow2\times2$

e AD cannot be formed

$3\times\boxed{2}\times\boxed{1}\times3$ - number of columns from matrix A $\neq$ number of rows in matrix D

f DA: $1\times\boxed{3}\times\boxed{3}\times2\Rightarrow1\times2$

g BCA: $\left(3\times\boxed{2}\times\boxed{2}\times3\right)\times3\times2\Rightarrow3\times\boxed{3}\times\boxed{3}\times2\Rightarrow3\times2$

h DAC: $\left(1\times\boxed{3}\times\boxed{3}\times2\right)\times2\times3\Rightarrow1\times\boxed{2}\times\boxed{2}\times3\Rightarrow1\times3$

Question 25

a AB: $1\times\boxed{2}\times\boxed{2}\times 1 \Rightarrow 1\times 1$ Yes

b BA: $2\times\boxed{1}\times\boxed{1}\times 2 \Rightarrow 2\times 2$ Yes

c AC: $1\times\boxed{2}\times\boxed{2}\times 2 \Rightarrow 1\times 2$ Yes

d CA cannot be formed

$2\times\boxed{2}\times\boxed{1}\times 2$ - number of columns from matrix C $\neq$ number of rows in matrix A

e BD cannot be formed

$2\times\boxed{1}\times\boxed{3}\times 1$ - number of columns from matrix B $\neq$ number of rows in matrix D

f DB cannot be formed

$3\times\boxed{1}\times\boxed{2}\times 1$ - number of columns from matrix D $\neq$ number of rows in matrix B

g AD cannot be formed

$1\times\boxed{2}\times\boxed{3}\times 1$ - number of columns from matrix A $\neq$ number of rows in matrix D

h DA: $3\times\boxed{1}\times\boxed{1}\times 2 \Rightarrow 3\times 2$ Yes

Question 26

If $\mathrm{A}_{a\times b}$, then $b = a$ which means A must be a square matrix

Question 27

$A_{2\times2}$ $B_{1\times2}$ $C_{2\times1}$

Possible products are :

AA AB AC BA BB BC CA CB CC

$A_{2\times2}A_{2\times2}$ is possible

$A_{2\times2}B_{1\times2}$ is not

$A_{2\times2}C_{2\times1}$ is possible

$B_{1\times2}A_{2\times2}$ is possible

$B_{1\times2}B_{1\times2}$ is not

$B_{1\times2}C_{2\times1}$ is possible

$C_{2\times1}A_{2\times2}$ is not

$C_{2\times1}B_{1\times2}$ is possible

$C_{2\times1}C_{2\times1}$ is not

Question 28

a

$$\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1\times2+(-1)\times3 & 1\times0+(-1)\times2 \\ 2\times2+0\times3 & 2\times0+0\times2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 \\ 4 & 0 \end{bmatrix}$$

b

$$\begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2\times1+0\times2 & 2\times(-1)+0\times0 \\ 3\times1+2\times2 & 3\times(-1)+2\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ 7 & -3 \end{bmatrix}$$

Question 29

a

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 0 \\ 0 & 3 & 3 \\ 1 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 18 \\ 12 \\ 11 \\ 13 \end{bmatrix}$$

Order from first to fifth: B, E, C, D, A

b

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 0 \\ 0 & 3 & 3 \\ 1 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ 15 \\ 15 \\ 10 \\ 14 \end{bmatrix}$$

Order from first to fifth : B and C, E, D, A

Question 30

Initial value of portfolio

$$\begin{array}{c} \text{Client1} \\ \text{Client2} \\ \text{Client3} \end{array} \begin{bmatrix} 1000 & 5000 & 400 & 270 \\ 500 & 8000 & 500 & 250 \\ 500 & 3000 & 500 & 500 \end{bmatrix} \begin{bmatrix} 5 \\ 0.5 \\ 12 \\ 10 \end{bmatrix} = \begin{array}{c} \text{Client1} \\ \text{Client2} \\ \text{Client3} \end{array} \begin{bmatrix} 15000 \\ 15000 \\ 15000 \end{bmatrix}$$

All portfolios are initially worth \$15 000

$$\begin{array}{c} \text{Client1} \\ \text{Client2} \\ \text{Client3} \end{array} \begin{bmatrix} 1000 & 5000 & 400 & 270 \\ 500 & 8000 & 500 & 250 \\ 500 & 3000 & 500 & 500 \end{bmatrix} \begin{bmatrix} 4 \\ 0.6 \\ 20 \\ 10 \end{bmatrix} = \begin{array}{c} \text{Client1} \\ \text{Client2} \\ \text{Client3} \end{array} \begin{bmatrix} 17700 \\ 19300 \\ 18800 \end{bmatrix}$$

After two years, Client 1’s portfolio is worth \$17 700, Client 2 worth \$19 300 and Client 3 is worth \$18 800.

Question 31

$$\text{Qty}\begin{bmatrix}15 & 10\end{bmatrix}\begin{bmatrix}375 & 1 \\ 1250 & 4\end{bmatrix} = \text{Qty}\begin{bmatrix}18125 & 55\end{bmatrix}$$

18.125L of drink and 55 burgers required to fill the order

Question 32

a $P_{3\times\boxed{3}}Q_{\boxed{1}\times 3}$ cannot be formed

$Q_{1\times\boxed{3}}P_{\boxed{3}\times 3}$ can be be formed

b $$QP = \begin{bmatrix}75 & 125 & 180\end{bmatrix}\begin{bmatrix}15 & 5 & 5 \\ 25 & 25 & 14 \\ 2 & 1 & 3\end{bmatrix}$$

$$= \begin{bmatrix}4610 & 3680 & 2665\end{bmatrix}$$

$$Q_{\text{cost per night} \times \text{S D S}}P_{\text{S D S} \times \text{HotelA B C}} = QP_{\text{cost per night} \times \text{Hotel A B C}}$$

QP shows income each night for each hotel if all rooms are occupied

$$R = \begin{bmatrix}75 \\ 125 \\ 180\end{bmatrix}$$

c $$PR = P_{\text{S D S} \times \text{Hotel A B C}}R_{\text{S D S} \times \text{cost per night}}$$

$$\begin{bmatrix}15 & 5 & 5 \\ 25 & 25 & 14 \\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}75 \\ 125 \\ 180\end{bmatrix} = \begin{bmatrix}15\times 75 + 5\times 125 + 5\times 180 \\ 25\times 75 + 25\times 125 + 14\times 80 \\ 2\times 75 + 1\times 125 + 3\times 180\end{bmatrix}$$

The first entry gives the total of the single rooms in Hotel A x cost of a single room added to the single rooms in Hotel B x the double room tariff added to the single rooms in Hotel C x by the suite tariff. This gives no useful information.

Question 33

a As matrix P has the cubby types as rows, the only way to get useful information is for the other matrix to have the cubby types listed as columns and to be premultiplied.

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \text{Number ordered} & \begin{bmatrix} 3 & 1 & 2 \end{bmatrix} \end{array}$$

b $$QP = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 30 & 20 & 40 \\ 4 & 35 & 25 & 60 \\ 6 & 40 & 30 & 70 \end{bmatrix} = \begin{bmatrix} 25 & 205 & 145 & 320 \end{bmatrix}$$

Number of metres required for the Poles, Decking, Framing and Sheeting
(i.e. the row label for matrix Q and the column label for matrix P

c $$R = \begin{bmatrix} 4 \\ 2 \\ 3 \\ 1.5 \end{bmatrix}$$

As matrix QP has the information for the four parts listed as column headers, matrix R will need them as row labels to be form a useful product.

R would have dimensions 3×1 and would show the cost per cubby.

Question 34

a As the Commodities are listed as rows in matrix D, matrix E would require them to be in columns. This means the cost matrix E is a row matrix, $E = \begin{bmatrix} 800 & 50 & 1000 \end{bmatrix}$

b The product required will be ED

$$ED = \begin{bmatrix} 800 & 50 & 1000 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 & 2 \\ 20 & 30 & 50 & 40 \\ 2 & 1 & 3 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 4600 & 4900 & 6300 & 5600 \end{bmatrix}$$

$$E_{\text{Cost} \times \text{CommodityABC}} D_{\text{CommodityABC} \times \text{Model1}-4}$$
$$= ED_{\text{Cost} \times \text{Models1-4}}$$

ED shows the total cost of commodities A, B and C required for each model.

Question 35

a As matrix P has the Models listed in rows, we require a matrix with model information listed in columns therefore matrix R is the sensible choice. Hence the produce RP is likely to be useful.

b $$\begin{bmatrix}50 & 100 & 80\end{bmatrix}\begin{bmatrix}30 & 20 & 10\\ 20 & 30 & 10\\ 40 & 40 & 10\end{bmatrix}=\begin{bmatrix}6700 & 7200 & 2300\end{bmatrix}$$

c) The first entry 6700 is formed as a result of $50\times 30+100\times 20+80\times 40$ showing the 50 model A's will need 1500 minutes in total in cutting, 100 model B will need 2000 minutes in cutting and 80 model C's will need 3200 minutes in cutting. This first number shows the total number of minutes need in the cutting area to fill this order.

The matrix shows the number of minutes required in each of the three areas to fill this order.

$$\begin{aligned}&Q_{\text{Number required}\times\text{ModelABC}}P_{\text{ModelABC}\times\text{Minutes of Cutting, Assembling and Pacing}}\\&=QP_{\text{Number required}\times\text{Minutes of Cutting, Assembling and Pacing}}\end{aligned}$$

Exercise 10C

Question 1

$1\times 4-2\times 3=-2$

Question 2

$4\times 1-(-2)\times 3=10$

Question 3

$(-1)\times(-1)-(-3)\times 2=7$

Question 4

$(-1)\times(-1)-3\times(-2)=7$

Question 5

$5\times 1-0\times 2=5$

Question 6

$1\times(-1)-1\times(-1)=0$

Question 7

$x\times(-x)-0\times y=-x^2$

Question 8

$$x \times x - y \times y = x^2 - y^2$$

Question 9

$$\frac{1}{2-1}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

Question 10

$$\frac{1}{9-8}\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$$

Question 11

$$\begin{aligned} \frac{1}{2-(-1)}\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix} &= \frac{1}{3}\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \end{aligned}$$

Question 12

$$\begin{aligned} \frac{1}{8-3}\begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix} &= \frac{1}{5}\begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 0.4 & -0.6 \\ -0.2 & 0.8 \end{bmatrix} \end{aligned}$$

Question 13

$$\frac{1}{9-(-1)}\begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 0.3 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$$

Question 14

$$\frac{1}{9-(-1)}\begin{bmatrix} -3 & -1 \\ 1 & -3 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} -3 & -1 \\ 1 & -3 \end{bmatrix}$$
$$= \begin{bmatrix} -0.3 & -0.1 \\ 0.1 & -0.3 \end{bmatrix}$$

Question 15

$ad - bc = 1 - 1 = 0$

Matrix is singular as the determinant is zero $\therefore$ no inverse exists

Question 16

$ad - bc = 24 - 24 = 0$

Matrix is singular as the determinant is zero $\therefore$ no inverse exists

Question 17

$ad - bc = 0 - 0 = 0$

Matrix is singular as the determinant is zero $\therefore$ no inverse exists

Question 18

$$\frac{1}{x-0}\begin{bmatrix} 1 & -y \\ 0 & x \end{bmatrix} = \frac{1}{x}\begin{bmatrix} 1 & -y \\ 0 & x \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{x} & -\frac{y}{x} \\ 0 & 1 \end{bmatrix}$$

Question 19

$$\frac{1}{1-0}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Question 20

$$\frac{1}{-1-0}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = -\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Question 21

a True

b True

c Not necessarily. Matrix multiplication is not commutative

d True

e True

f True

g True

h True

i Not necessarily. The null factor law does not apply to matrices.

j Not necessarily

Question 22

$$\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} A = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} A = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Question 23

$$\begin{bmatrix} 5 & 1 \\ 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0.5 & -0.5 \\ -1.5 & 2.5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ 3 & 1 \end{bmatrix} \mathrm{B} = \begin{bmatrix} 9 \\ 5 \end{bmatrix} -$$

$$\begin{bmatrix} 0.5 & -0.5 \\ -1.5 & 2.5 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 3 & 1 \end{bmatrix} \mathrm{B} = \begin{bmatrix} 0.5 & -0.5 \\ -1.5 & 2.5 \end{bmatrix}\begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

$$\mathrm{B} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Question 24

$$\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0.1 & 0.3 \\ -0.2 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix} \mathrm{C} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 0.1 & 0.3 \\ -0.2 & 0.4 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix} \mathrm{C} = \begin{bmatrix} 0.1 & 0.3 \\ -0.2 & 0.4 \end{bmatrix}\begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$$\mathrm{C} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

Question 25

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \mathrm{D} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \mathrm{D} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\mathrm{D} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Question 26

$$\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix} - 2\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 8 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 6 & 8 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 7\mathrm{I}$$

$$\Rightarrow k = 7$$

Question 27

$$\begin{bmatrix} k & -2 \\ 5 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0.2 \\ -0.5 & 0.1k \end{bmatrix}$$

$$\begin{bmatrix} k & -2 \\ 5 & 0 \end{bmatrix} + 10\begin{bmatrix} 0 & 0.2 \\ -0.5 & 0.1k \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\Rightarrow k = 5$$

Question 28

a $\begin{bmatrix} -13 & 4 \\ 12 & -4 \end{bmatrix}$

b $16 \times 5 - (-5) \times (-14) = 10$

c $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

d $\begin{bmatrix} 0.5 & 0.5 \\ 1.4 & 1.6 \end{bmatrix}$

e
$$\text{AC} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$\begin{aligned} \text{A}^{-1}\text{AC} &= \text{A}^{-1}\begin{bmatrix} 9 \\ -5 \end{bmatrix} \\ \text{C} &= \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ -5 \end{bmatrix} \\ &= \begin{bmatrix} 4 \\ 3 \end{bmatrix} \end{aligned}$$

f
$$\begin{aligned} \text{DA} &= \text{B} \\ \text{DAA}^{-1} &= \text{BA}^{-1} \\ \text{D} &= \text{BA}^{-1} \\ &= \begin{bmatrix} 16 & -5 \\ -14 & 5 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 1 \\ -4 & 1 \end{bmatrix} \end{aligned}$$

Question 29

a $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

b $\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$

c $\begin{bmatrix} \frac{1}{6} & -\frac{1}{3} \\ 0 & 1 \end{bmatrix}$

d $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

e
$$\begin{aligned} R(P+Q) &= Q \\ R(P+Q)(P+Q)^{-1} &= Q(P+Q)^{-1} \\ R &= \begin{bmatrix} 6 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 6 & -2 \\ -3 & 2 \end{bmatrix} \end{aligned}$$

Question 30

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 1 \\ 22 & 7 \end{bmatrix} \\ ABB^{-1} &= \begin{bmatrix} 3 & 1 \\ 22 & 7 \end{bmatrix}B^{-1} \\ A &= \begin{bmatrix} 3 & 1 \\ 22 & 7 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -1 \\ 17 & -9 \end{bmatrix} \end{aligned}$$

Question 31

$$\mathrm{CD} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

$$\mathrm{C}^{-1}\mathrm{CD} = \mathrm{C}^{-1}\begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

$$\mathrm{D} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Question 32

a $3x - 24 = 0$

$3x = 24$

$x = 8$

b $x^2 - 16 = 0$

$x^2 = 16$

$x = \pm 4$

c $x(x-1) - 20 = 0$

$x^2 - x - 20 = 0$

$(x+4)(x-5) = 0$

$x = -4, x = 5$

Question 33

$$EF = \begin{bmatrix} -2 & 12 \\ 0 & 9 \end{bmatrix}$$

$$E^{-1}EF = E^{-1}\begin{bmatrix} -2 & 12 \\ 0 & 9 \end{bmatrix}$$

$$F = \begin{bmatrix} -0.5 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -2 & 12 \\ 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 2 & -3 \end{bmatrix}$$

$$GE = \begin{bmatrix} -2 & -2 \\ 4 & -2 \end{bmatrix}$$

$$GEE^{-1} = \begin{bmatrix} -2 & -2 \\ 4 & -2 \end{bmatrix}E^{-1}$$

$$G = \begin{bmatrix} -2 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} -0.5 & 1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 \\ 0 & 2 \end{bmatrix}$$

Question 34

$$AC = B$$

$$A^{-1}AC = A^{-1}B$$

$$C = A^{-1}B$$

$$A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 3 & -2 \\ 7 & -10 & 8 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 3 & -2 \\ 7 & -10 & 8 \end{bmatrix}\begin{bmatrix} -1 & -6 & -11 \\ 4 & 1 & 1 \\ 6 & 7 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 1 & 4 & 1 \end{bmatrix}$$

Question 35

$$CA = B$$
$$CAA^{-1} = BA^{-1}$$
$$A^{-1} = \begin{bmatrix} 5 & -9 & 3 \\ -2 & 4 & -1 \\ 1 & -2 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 4 & 6 & -7 \\ 1 & 5 & 5 \\ 7 & 11 & -10 \end{bmatrix}\begin{bmatrix} 5 & -9 & 3 \\ -2 & 4 & -1 \\ 1 & -2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 3 & 1 & 0 \end{bmatrix}$$

Question 36

a $\begin{bmatrix} \$24 & \$56 \\ \$16 & \$36 \end{bmatrix}$

b

$$CA = B$$
$$CAA^{-1} = BA^{-1}$$
$$C = BA^{-1}$$

$$A^{-1} = \begin{bmatrix} -0.9 & 1.75 \\ 0.4 & -0.75 \end{bmatrix}$$
$$C = \begin{bmatrix} \$24 & \$56 \\ \$16 & \$36 \end{bmatrix}\begin{bmatrix} -0.9 & 1.75 \\ 0.4 & -0.75 \end{bmatrix}$$
$$= \begin{bmatrix} 0.8 & 0 \\ 0 & 1 \end{bmatrix}$$

Question 37

$$C = A\text{-}CB$$

$$C + CB = A$$

$$CI + CB = A$$

$$C(I + B) = A$$

$$C(I + B)(I + B)^{-1} = A(I + B)^{-1}$$

$$C = A(I + B)^{-1}$$

$$C = A(I + B)^{-1}$$

$$= \begin{bmatrix} -1 & 6 \\ 11 & 4 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix} \right)^{-1}$$

$$= \begin{bmatrix} -1 & 6 \\ 11 & 4 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -5 & 2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -1 & 6 \\ 11 & 4 \end{bmatrix} \frac{1}{4+10} \begin{bmatrix} 2 & -2 \\ 5 & 2 \end{bmatrix}$$

$$= \frac{1}{14} \begin{bmatrix} -1 & 6 \\ 11 & 4 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 5 & 2 \end{bmatrix}$$

$$= \frac{1}{14} \begin{bmatrix} 28 & 14 \\ 42 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$$

Question 38

$$\begin{aligned} A &= BC - AC \\ &= (B-A)C \\ (B-A)^{-1}A &= (B-A)^{-1}(B-A)C \\ C &= (B-A)^{-1}A \end{aligned}$$

$$\begin{aligned} C &= (B-A)^{-1}A \\ &= \left(\begin{bmatrix} -1 & 0 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 5 \\ 1 & 6 \end{bmatrix}\right)^{-1} \begin{bmatrix} -3 & 5 \\ 1 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}^{-1} \begin{bmatrix} -3 & 5 \\ 1 & 6 \end{bmatrix} \\ &= \frac{1}{-4+5}\begin{bmatrix} -2 & 5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 1 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 11 & 20 \\ 5 & 7 \end{bmatrix} \end{aligned}$$

Question 39

$$\begin{aligned} &P = Q + PQ + PQ^2 \\ &P - PQ - PQ^2 = Q \\ &P(I - Q - Q^2) = Q \\ &P = Q(I - Q - Q^2)^{-1} \end{aligned}$$

$$\begin{aligned} P &= \begin{bmatrix} -1 & 0 \\ 5 & -2 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 5 & -2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 5 & -2 \end{bmatrix}^2\right)^{-1} \\ &= \begin{bmatrix} -1 & 0 \\ 5 & -2 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 5 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -15 & 4 \end{bmatrix}\right)^{-1} \\ &= \begin{bmatrix} -1 & 0 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 10 & -1 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} -1 & 0 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 10 & -1 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 0 \\ -15 & 2 \end{bmatrix} \end{aligned}$$

Question 40

a $\begin{bmatrix} 6 & 5 \\ 8 & 7 \end{bmatrix}$

b $\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 6 & 5 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 860 & 740 \end{bmatrix}$

$\text{BA} = \begin{bmatrix} 860 & 740 \end{bmatrix}$

c

$$\begin{aligned}
\text{BA} &= \begin{bmatrix} 860 & 740 \end{bmatrix} \\
\text{BAA}^{-1} &= \begin{bmatrix} 860 & 740 \end{bmatrix}\text{A}^{-1} \\
\text{B} &= \begin{bmatrix} 860 & 740 \end{bmatrix}\frac{1}{42-40}\begin{bmatrix} 7 & -5 \\ -8 & 6 \end{bmatrix} \\
\begin{bmatrix} x & y \end{bmatrix} &= \frac{1}{2}\begin{bmatrix} 860 & 740 \end{bmatrix}\begin{bmatrix} 7 & -5 \\ -8 & 6 \end{bmatrix} \\
&= \begin{bmatrix} 50 & 70 \end{bmatrix}
\end{aligned}$$

Exercise 10D

Question 1

$$\begin{bmatrix} 2 & 3 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

Question 2

$$\begin{bmatrix} -1 & 2 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

Question 3

$$\begin{bmatrix} 3 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Question 4

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -4 & 2 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

Question 5

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 0 \\ 2 & 0 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 5 \\ 4 \\ 0 \end{bmatrix}$$

Question 6

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & -3 \\ 0 & -2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

Question 7

a

$$\begin{aligned} \mathrm{A}^{-1} &= \frac{1}{12-10}\begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 1 \\ 2.5 & 1.5 \end{bmatrix} \end{aligned}$$

b

$$\begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -9 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 2.5 & 1.5 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2.5 & 1.5 \end{bmatrix}\begin{bmatrix} 4 \\ -9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -3.5 \end{bmatrix}$$

$x = -1, y = -3.5$

Question 8

a $A^{-1}=\begin{bmatrix}-2.5 & -2 & 0.5\\ -2 & -2 & 1\\ 1 & 1 & 0\end{bmatrix}$

b $$\begin{bmatrix}-2 & 1 & -2\\ 2 & -1 & 3\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ -1\\ 9\end{bmatrix}$$

$$\begin{bmatrix}-2.5 & -2 & 0.5\\ -2 & -2 & 1\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}-2 & 1 & -2\\ 2 & -1 & 3\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-2.5 & -2 & 0.5\\ -2 & -2 & 1\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}3\\ -1\\ 9\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-2.5 & -2 & 0.5\\ -2 & -2 & 1\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}3\\ -1\\ 9\end{bmatrix}$$

$$=\begin{bmatrix}-1\\ 5\\ 2\end{bmatrix}$$

$x=-1, y=5, z=2$

Question 9

a
$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{6-5}\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$=\begin{bmatrix} 3 \\ -7 \end{bmatrix}$$

$x=3, y=-7$

b
$$\begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 8 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 8 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{9-7}\begin{bmatrix} 3 & -1 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} 8 \\ 13 \end{bmatrix}$$

$$=\begin{bmatrix} 5.5 \\ -8.5 \end{bmatrix}$$

$x=5.5, y=-8.5$

Question 10

a

$$\begin{bmatrix} -2 & -1 & 2 \\ 1 & -1 & 3 \\ 3 & 2 & -2 \end{bmatrix}\begin{bmatrix} -4 & 2 & -1 \\ 11 & -2 & 8 \\ 5 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

b

$$\begin{aligned} AB &= 7I \\ A^{-1}AB &= 7A^{-1}I \\ B &= 7A^{-1} \\ A^{-1} &= \frac{1}{7}B \end{aligned}$$

c

$$\begin{bmatrix} -2 & -1 & 2 \\ 1 & -1 & 3 \\ 3 & 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix}$$

$$\begin{aligned} A\begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix} \\ A^{-1}A\begin{bmatrix} x \\ y \\ z \end{bmatrix} &= A^{-1}\begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix} \\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \frac{1}{7}B\begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix} \\ &= \frac{1}{7}\begin{bmatrix} -4 & 2 & -1 \\ 11 & -2 & 8 \\ 5 & 1 & 3 \end{bmatrix}\begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \end{aligned}$$

$x = 3, y = -1, z = 1$

Question 11

a $$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 2 & -1 \\ 2 & -1 & 3 & -1 & 2 \\ 3 & 2 & -1 & -1 & -2 \\ 0 & 2 & 0 & 3 & -1 \end{bmatrix}\begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 13 \\ 2 \\ 4 \\ 8 \end{bmatrix}$$

b $$A^{-1} = \begin{bmatrix} \frac{56}{71} & \frac{25}{71} & -\frac{23}{71} & \frac{12}{71} & -\frac{39}{71} \\ -\frac{24}{71} & -\frac{31}{71} & \frac{20}{71} & \frac{5}{71} & \frac{37}{71} \\ -\frac{85}{71} & -\frac{24}{71} & \frac{59}{71} & -\frac{3}{71} & \frac{63}{71} \\ \frac{43}{71} & \frac{23}{71} & -\frac{24}{71} & -\frac{6}{71} & -\frac{16}{71} \\ \frac{81}{71} & \frac{7}{71} & -\frac{32}{71} & -\frac{8}{71} & -\frac{45}{71} \end{bmatrix}$$

$$\begin{aligned} AX &= B \\ X &= A^{-1}B \\ &= \begin{bmatrix} \frac{56}{71} & \frac{25}{71} & -\frac{23}{71} & \frac{12}{71} & -\frac{39}{71} \\ -\frac{24}{71} & -\frac{31}{71} & \frac{20}{71} & \frac{5}{71} & \frac{37}{71} \\ -\frac{85}{71} & -\frac{24}{71} & \frac{59}{71} & -\frac{3}{71} & \frac{63}{71} \\ \frac{43}{71} & \frac{23}{71} & -\frac{24}{71} & -\frac{6}{71} & -\frac{16}{71} \\ \frac{81}{71} & \frac{7}{71} & -\frac{32}{71} & -\frac{8}{71} & -\frac{45}{71} \end{bmatrix}\begin{bmatrix} 1 \\ 13 \\ 2 \\ 4 \\ 8 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -1 \\ 3 \\ 2 \\ -4 \end{bmatrix} \end{aligned}$$

$v = 1, w = -1, x = 3, y = 2$ and $z = -4$

Question 1

a

$$\begin{bmatrix} 2 & 0 \\ -4 & -3 \end{bmatrix} + \mathrm{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathrm{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ -4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 \\ 4 & 3 \end{bmatrix}$$

b

$$\mathrm{A} + \mathrm{C} = \mathrm{I}$$

$$\mathrm{C} = \mathrm{I} - \mathrm{A}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ -4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 4 & 4 \end{bmatrix}$$

Question 2

a $$E = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

b $$F = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$

c $$G = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 6 & -1 \\ 2 & 1 \end{bmatrix}$$

d $$H = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$

e $$K = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$

Question 3

$$\sin(x + \frac{\pi}{4}) = \frac{\sqrt{3}}{2}$$

$$0 \le x \le \pi \quad \Rightarrow \quad \frac{\pi}{4} \le x + \frac{\pi}{4} \le \frac{5\pi}{4}$$

$$(x + \frac{\pi}{4}) = \frac{\pi}{3}, \frac{2\pi}{3}$$
$$x = \frac{\pi}{12}, \frac{5\pi}{12}$$

Question 4

$$k\sin\theta = 2\sin^2\theta \qquad 0 \le \theta \le 2\pi$$
$$2\sin^2\theta - k\sin\theta = 0$$
$$2\sin\theta(\sin\theta - k\cos\theta) = 0$$
$$2\sin\theta = 0 \text{ or } \sin\theta - k\cos\theta = 0$$
$$\sin\theta = 0 \qquad \sin\theta = k\cos\theta$$
$$\theta = 0, \pi, 2\pi \qquad \tan\theta = k$$
$$\theta = p, p+\pi$$
$$\theta = 0, p, \pi, p+\pi, 2\pi$$

Question 5

$$\begin{aligned}\text{LHS} &= 2\sin^3\theta\cos\theta + 2\cos^3\theta\sin\theta \\ &= 2\sin\theta\cos\theta(\sin^2\theta + \cos^2\theta) \\ &= 2\sin\theta\cos\theta \times 1 \\ &= \sin 2\theta \\ &= \text{RHS}\end{aligned}$$

Question 6

$$\begin{aligned}\text{LHS} &= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} \times \frac{(\cos\theta - \sin\theta)}{(\cos\theta - \sin\theta)} \\ &= \frac{\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} \\ &= \frac{1 - 2\sin\theta\cos\theta}{\cos^2\theta - (1 - \cos^2\theta)} \\ &= \frac{1 - \sin 2\theta}{2\cos^2\theta - 1} \\ &= \frac{1 - \sin 2\theta}{\cos 2\theta} \\ &= \text{RHS}\end{aligned}$$

Question 7

a $(2y-1)(y+1)=2y^2+y-1$

b

$$1+\sin x=2\cos^2 x$$
$$1+\sin x=2(1-\sin^2 x)$$
$$2\sin^2 x-2+\sin x+1=0$$
$$2\sin^2 x+\sin x-1=0$$
$$(2\sin x-1)(\sin x+1)=0$$

$2\sin x-1=0$ or $\sin x+1=0$

$\sin x=\frac{1}{2}$ $\quad$ $\sin x=-1$

$x=\frac{-11\pi}{6},\frac{-7\pi}{6},\frac{\pi}{6},\frac{5}{6}$ $\quad$ $x=-\frac{\pi}{2},\frac{3\pi}{2}$

$$x=\frac{-11\pi}{6},\frac{-7\pi}{6}-\frac{\pi}{2},\frac{\pi}{6},\frac{5\pi}{6}\frac{3\pi}{2}$$

Question 8

a $R = \sqrt{2^2 + 5^2} = \sqrt{29}$

$$\sqrt{29}\left(\frac{2}{\sqrt{29}}\cos\theta + \frac{5}{\sqrt{29}}\sin\theta\right) = R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$$

$$\cos\alpha = \frac{2}{\sqrt{29}} \text{ \& } \sin\alpha = \frac{5}{\sqrt{29}}$$

$$\tan\alpha = \frac{5}{2}$$

$$\alpha = 68.2°$$

$$2\cos\theta + 5\sin\theta = \sqrt{29}\cos(\theta - 68.2°)$$

b Minimum value of $\cos(\theta - \alpha) = -1$ therefore the minimum value of $2\cos\theta + 5\sin\theta = -\sqrt{29}$.

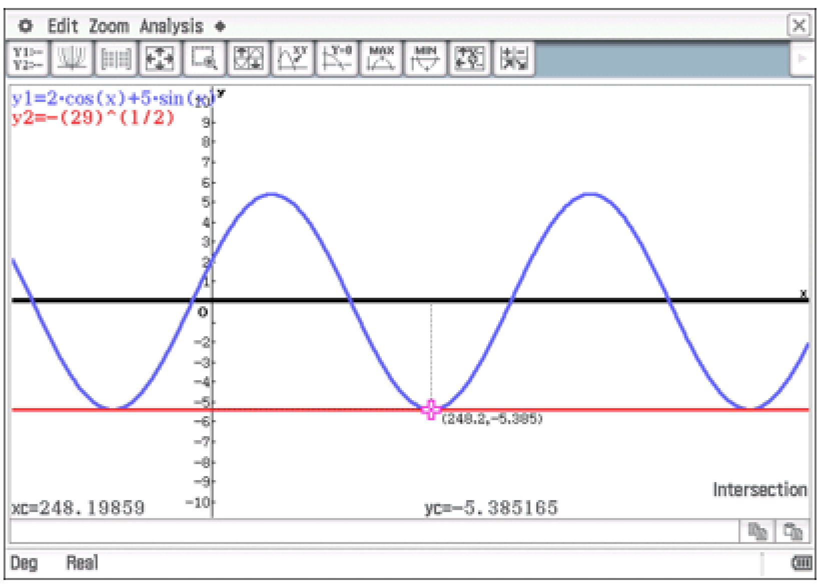
$$\sqrt{29}\cos(\theta - 68.2°) = -\sqrt{29}$$

$$\cos(\theta - 68.2°) = -1$$

$$\theta - 68.2° = 180°$$

$$\theta = 248.2°$$

c



Question 9

a Dimensions of A and B are not equal (i.e. they are not the same size) and cannot be added.

b
$$2\begin{bmatrix}1 & 0 & 2\\ 2 & -1 & 3\end{bmatrix}-\begin{bmatrix}2 & 1 & 3\\ -1 & 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}2 & 0 & 4\\ 4 & -2 & 6\end{bmatrix}-\begin{bmatrix}2 & 1 & 3\\ -1 & 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}0 & -1 & 1\\ 5 & -2 & 5\end{bmatrix}$$

c Cannot be determined as the number of columns in A is not the same as the number of rows in B.

d
$$\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 2 & -1 & 3\end{bmatrix}=\begin{bmatrix}5 & -2 & 8\\ 2 & -1 & 3\end{bmatrix}$$

e Cannot be determined as the number of columns in A is not the same as the number of rows in C.

f
$$\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1\\ 2\end{bmatrix}=\begin{bmatrix}5\\ 2\end{bmatrix}$$

Question 10

a Given matrix X has listed PQR information in rows, we need cost information for PQR to be listed as a column matrix, so XY is the useful product.

b
$$\begin{bmatrix}2 & 2 & 1\\ 3 & 1 & 1\\ 1 & 3 & 1\end{bmatrix}\begin{bmatrix}50\\ 60\\ 200\end{bmatrix}=\begin{bmatrix}2\times 50+2\times 60+1\times 200\\ 3\times 50+1\times 60+1\times 200\\ 1\times 50+3\times 60+1\times 200\end{bmatrix}=\begin{bmatrix}420\\ 410\\ 430\end{bmatrix}$$

c Cost of the commodities P, Q and R required for each model.

Question 11

$X_{5\times3}$ $Y_{1\times3}$ $Z_{3\times1}$

Comparison of numbers of rows and columns shows the only product which can be made is XZ.

$$\begin{bmatrix} 4 & 1 & 3 \\ 3 & 1 & 4 \\ 2 & 3 & 3 \\ 3 & 1 & 4 \\ 5 & 0 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 13 \\ 10 \\ 9 \\ 10 \\ 15 \end{bmatrix}$$

Question 12

a Graph completes 2 cycles in $\frac{\pi}{2}$ therefore in 2π it will complete 8 $\Rightarrow b = 8$

The amplitude is 4.

$y = 4\sin 8x$

b The amplitude is 3 but *a* will be negative due to the shape of the graph

The period of the wave is 5. $\frac{2\pi}{b} = 5 \Rightarrow b = \frac{2\pi}{5}$

$$y = -3\sin\left(\frac{2\pi}{5}x\right)$$

Question 13

a The wave has been shifted right by 1 unit so $c = -1$.

The amplitude of the wave is 2, giving $a = 2$.

The period of the wave is 4 ; $\frac{2\pi}{b} = 4 \Rightarrow b = \frac{2\pi}{4} = \frac{\pi}{2}$.

$$y = 2\sin\left(\frac{\pi}{2}(x-1)\right)$$

b The curve has been shifted 25 units right at first glance.

The period is (25 -10) x 2 =30; $\frac{2\pi}{b} = 30 \Rightarrow b = \frac{2\pi}{30} = \frac{\pi}{15}$

We can use this information to decide the shift of the curve could also be expressed as

5 units to the left.

The amplitude of the wave is 20.

$$y = 20\sin\left(\frac{\pi}{15}(x+5)\right)$$

Question 14

a Mean or central value is 10 therefore d =10.

The amplitude is 5 giving a = 5.

The curve has been shifted 2 units right so c = –2.

The period of the curve is 10; $\frac{2\pi}{b} = 10 \Rightarrow b = \frac{2\pi}{10} = \frac{\pi}{5}$

$$y = 5\sin\left(\frac{\pi}{5}(x-2)\right) + 10$$

b The amplitude is 10; $a = \frac{50-30}{2} = 10$.

The mean value is $d = \frac{50+30}{2} = 40$.

The period is (94 -45) x 2 =100; $\frac{2\pi}{b} = 100 \Rightarrow b = \frac{2\pi}{100} = \frac{\pi}{50}$

The maximum value for a sin curve occurs one quarter of the way through the wave.

The wave must then start when x = 20. This gives c = –20.

$$y = 10\sin\left(\frac{\pi}{50}(x-20)\right) + 40$$

Question 15

$$\mathrm{AB}=\begin{bmatrix} x & 2 \\ y & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}=\begin{bmatrix} 3x-2 & x+8 \\ 3y-1 & y+4 \end{bmatrix}$$

$$\mathrm{BA}=\begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} x & 2 \\ y & 1 \end{bmatrix}=\begin{bmatrix} 3x+y & 7 \\ -x+4y & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3x-2 & x+8 \\ 3y-1 & y+4 \end{bmatrix}=\begin{bmatrix} 3x+y & 7 \\ -x+4y & 2 \end{bmatrix}=\begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$\begin{aligned} &q=7, s=2 \\ &x+8=7 \Rightarrow x=-1 \\ &y+4=2 \Rightarrow y=-2 \end{aligned}$$

$$\begin{aligned} p &= 3x+y \\ &= 3(-1)+2 \\ &= -5 \\ r &= 3y-1 \\ &= 3(-2)-1 \\ &= -7 \end{aligned}$$

Question 16

$$\begin{aligned} &\mathrm{AB}+\mathrm{BP}+\mathrm{P}=\mathrm{Q} \\ &(\mathrm{A}+\mathrm{B}+\mathrm{I})\mathrm{P}=\mathrm{Q} \\ &\mathrm{P}=(\mathrm{A}+\mathrm{B}+\mathrm{I})^{-1}\mathrm{Q} \end{aligned}$$

$$\mathrm{A}+\mathrm{B}+\mathrm{I}=\begin{bmatrix} 8 & 2 \\ 7 & 2 \end{bmatrix}$$

$$\begin{aligned} (\mathrm{A}+\mathrm{B}+\mathrm{I})^{-1} &= \frac{1}{16-14}\begin{bmatrix} 2 & -2 \\ -7 & 8 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 2 & -2 \\ -7 & 8 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \mathrm{P} &= (\mathrm{A}+\mathrm{B}+\mathrm{I})^{-1}\mathrm{Q} \\ &= \frac{1}{2}\begin{bmatrix} 2 & -2 \\ -7 & 8 \end{bmatrix}\begin{bmatrix} -2 & -2 \\ -1 & -3 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 1 \\ 3 & -5 \end{bmatrix} \end{aligned}$$

Question 17

a

$$AB = BA$$
$$ABB^{-1} = BAB^{-1}$$
$$A = BAB^{-1}$$
$$B^{-1}A = B^{-1}BAB^{-1}$$
$$B^{-1}A = AB^{-1}$$

A and B^{-1} are commutative for multiplication

b

$$AB = BA$$
$$A^{-1}AB = A^{-1}BA$$
$$B = A^{-1}BA$$
$$BA^{-1} = A^{-1}BAA^{-1}$$
$$BA^{-1} = A^{-1}B$$

A^{-1} and B are commutative for multiplication